GDYNIA MARITIME UNIVERSITY
FACULTY OF MECHANICAL

PHYSICS LABORATORY
DEPARTMENT OF PHYSICS

Theoretical Introduction

Experiment "S T E I N E R"

The moment of inertia is a scalar physical quantity that depends on the distribution of mass relative to the axis of rotation. According to Newton's second law of dynamics for the rotational motion of a rigid body, the angular acceleration $\varepsilon$is directly proportional to the resultant torque M, and inversely proportional to the moment of inertia I.

$$\varepsilon = \frac{M}{I}$$

The moment of inertia I is therefore a measure of the inertia of a body in rotational motion and plays a role analogous to mass in the dynamics of translational motion.

The moment of inertia for a single material point of mass m, rotating at a distance r from the axis of rotation, is:

$$I = m \cdot r^2$$

The moment of inertia for a rigid body consisting of $n$connected material points is the sum:

$$I = \sum_{i=1}^{n} m_i r_i^2$$

The moment of inertia for a rigid body with a continuous mass distribution and uniform density ρ. Here, summation is replaced by integration over the entire volume of the body:

$$I = \int r^2 \mathrm{dm} = \rho \int r^2 \mathrm{dV}$$

The operational, differential, definition of the moment of inertia is as follows: the contribution to the moment of inertia from an elementary, infinitesimally small, mass dm, located at a distance $r$from the axis of rotation, is the product of the square of this distance and the elementary mass:

$$dI = r^2 dm$$

Steiner's Parallel Axis Theorem. If we know the moment of inertia of a body $I_0$with respect to an axis of rotation passing through the centre of mass of this body, then the moment of inertia $I$with respect to a new axis, parallel to the original one, can be written as:

$$I = I_0 + \mathrm{mb}^2$$

where:
m– mass of the body, b– distance between the axes of rotation.

Physical Pendulum. A physical pendulum is any rigid body suspended on a horizontal axis of rotation, which performs oscillations in a gravitational field. A pendulum displaced from its equilibrium position by an angle θ is acted upon by the torque M:

$$M = -mg \cdot d \cdot \sin\theta,$$

where:
m– mass of the pendulum, g– gravitational acceleration, d– distance from the axis of rotation to the centre of mass.
In the approximation of small oscillations: (*sin* θ ≈ θ), the period of oscillation of the pendulum is given by the formula:

$$T = 2\pi\sqrt{\frac{I}{mgd}}.$$

Torsion Pendulum. A torsion pendulum performs harmonic oscillations in torsional motion, usually around a vertical axis of rotation. Torsional harmonic motion occurs when a body is acted upon by a torque M, proportional to the angular displacement $\varphi$from the equilibrium position:

$$M = -D\varphi,$$

D - torsional elasticity modulus, related to the elasticity coefficient of the material from which the spring is made and to its geometrical dimensions. The period of oscillation of a torsion pendulum, by analogy with a physical pendulum, is:

$$T = 2\pi\sqrt{\frac{I}{D}}$$

“S T E I N E R”

Preparation Questions:

1. Give the definition and unit of the moment of inertia.
2. Give the definition and unit of torque.
3. Discuss the principles of dynamics for the rotational motion of a rigid body.
4. Give the formulas for the moments of inertia of basic rigid bodies: cylinder, disc, rod and sphere.
5. State Steiner’s theorem.
6. What is a physical pendulum?
7. What is a torsion pendulum?
8. Discuss the operating principle of a torque wrench.
9. What is the period of oscillation of a pendulum?
10. What is the frequency of oscillation of a pendulum?
11. Discuss the principle of conservation of energy in rotational motion.
12. Give the formula for the kinetic energy of rotational motion of a rigid body.
13. How do the amplitude and period of oscillations change in damped oscillatory motion?
14. Calculate the moment of inertia of a cylinder with a radius of 1 m and a mass of 100 kg when rotating around its edge.
15. Calculate the moment of inertia of a rod with a length of 1 m and a mass of 100 g when rotating around its end.

Instructions for performing the measurements:

1. First, examine the measuring set in order to be able to assemble it correctly in the later steps.
2. Before starting the actual measurements, remove the disc with the angular scale from the mount in order to perform static measurements of the disc, mass and radius.
3. Measure the mass of the disc using a scale, after first zeroing it, using the „TARE” button.
4. Measure the radius, or diameter, of the disc using the ruler available at the workstation.
5. Place the disc by its central hole on the stand equipped with a strip spring. The disc should be positioned in such a way that the end of the marker is located near the time-measuring gate. Remember to tighten the disc firmly.
6. Deflect the mounted disc by 90 degrees, press the „SET” button on the time-measuring gate and release the disc in order to measure one period. Record the result in the first column of the results table, entering the measured time and the value $b = 0$.
7. Remove the disc again in order to measure the distance of the next hole from the axis of the disc b, and mount the disc in this hole analogously to point 5.
8. Perform measurements of the periods T for the remaining distances $b$between its centre and the axis of rotation.
9. Estimate the measurement uncertainties:
   ΔT – for the greatest value of b, repeat the measurement several times, observing the spread of the results,
   Δb – uwzglednić dokładność przyrządu i dokładnosć wykonania otworów.
10. After completing the measurements, tidy up the workstation.
11. Perform the calculations according to the formulas in the report. In the calculations, remember to use the basic SI units.

“S T E I N E R”

Methodological Template

Student 1: Determination of the moment of inertia of a body using the dynamic method.
Student 2: Verification of Steiner’s theorem.

Theoretical Basis

The period of oscillation of a torsion pendulum is:

$$T = 2\pi\sqrt{\frac{I}{D}},$$

where the moment of inertia I, according to Steiner’s theorem, is:

$$I = I_0 + \mathrm{mb}^2$$

After substitution and squaring:

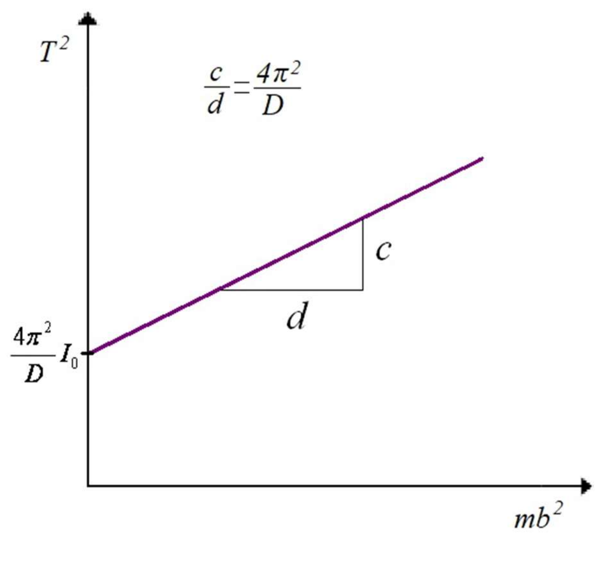
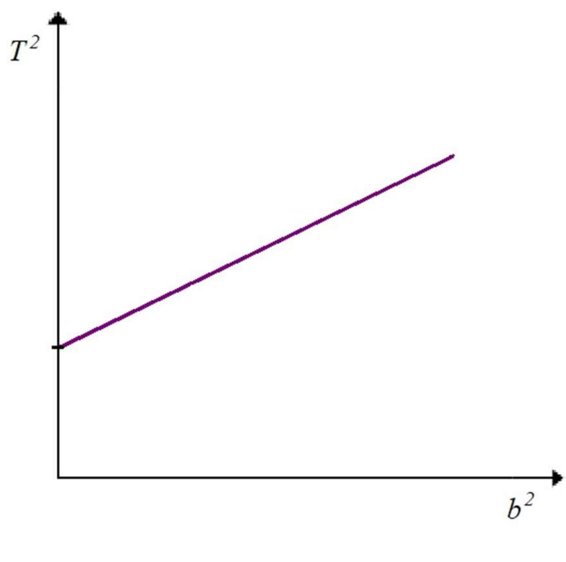
$$T^2 = \frac{4\pi^2}{D}(I_0 + \mathrm{mb}^2)$$

This can be represented as a linear function:

$$Y = A\,X + B:$$

$$\underbrace{T^2}_{Y} = \underbrace{\frac{4\pi^2}{D}}_{A}\underbrace{mb^2}_{X} + \underbrace{\frac{4\pi^2}{D}I_0}_{B}$$

The required moment of inertia is: $I_0 = \frac{B}{A}$.

	
Therefore, in order to determine the moment of inertia of the disc using a torsion pendulum, one should: • measure the period of torsional oscillations of the disc depending on the distance b between its centre and the axis of rotation, • prepare a graph of the dependence: T^2 as a function of: mb^2, • determine the moment of inertia of the disc $I_0$from the parameters of the straight line, as the intercept coefficient Bdivided by the slope coefficient A.	Therefore, in order to verify Steiner’s theorem, one should: • measure the period of torsional oscillations of the disc depending on the distance $b$between its centre and the axis of rotation, • prepare a graph of the dependence: T^2 as a function of: b^2, • analyse its linearity.

"S T E I N E R"

Report Guidelines – Determination

Student 1: Determination of the moment of inertia of a body using the dynamic method.

I. **Methodology** (conceptual plan of the exercise)
II. **Course of the exercise**
 II.1. Course of activities
 II.2. Sketch of the measurement setup
III. **Results**

III.1. Measurement results

		1	2	3	4	5	6	7	8	9	10
T	[s]										
b	[m]										

ΔT = ... R = ... m = ...
Δb = ... ΔR = ... Δm = ...

III.2. Calculations (example calculations - referring to measurement no. 3)

T^2 = ...
mb^2 = ...
$\Delta T^2 = | T^2 - (T + \Delta T)^2 |$ = ...
$\Delta mb^2 = b^2 \Delta m + 2mb \Delta b$ = ...

III.3. Calculation results for the graph

		1	2	3	4	5	6	7	8	9	10
T^2	[...]										
mb^2	[...]										
ΔT^2	[...]										
Δmb^2	[...]										

III.4. Graph
On which the calculations of the slope coefficient $A$and the intercept coefficient $B$of the straight line in the form Y = A·X + B) are shown:

- calculation of I_0, as the quotient $B/A$for the "best-fit" straight line
- calculation of I_0', as the quotient $B'/A'$for the deviated straight line
- calculation of: $\Delta I_0 = |I_0 - I_0'|$

IV. SUMMARY

The value of the moment of inertia I_0determined using the dynamic method is …
Accuracy of the method: …
For comparison, the value of the moment of inertia of the disc calculated using the static method:
$I_0 = 0{,}5\, m R^2$ is …
The differences between the moments of inertia calculated using these methods result from …
Additional conclusions, observations, causes of measurement uncertainties.

"S T E I N E R"

Report Guidelines – Verification

Student 2: Verification of Steiner's theorem.

I. **Methodology** (conceptual plan of the exercise)
II. **Course of the exercise**
  II.1. Course of activities
  II.2. Sketch of the measurement setup
III. **Results**

III.1. Measurement results

		1	2	3	4	5	6	7	8	9	10
T	[s]										
b	[m]										

ΔT = ...
Δb = ...

III.2. Calculations (example calculations - referring to measurement no. 3

T^2 = ...
b^2 = ...
$\Delta T^2 = | T^2 - (T + \Delta T)^2 | =$ ...
$\Delta b^2 = | b^2 - (b + \Delta b)^2 | =$ ...

III.3. Calculation results

		1	2	3	4	5	6	7	8	9	10
T^2	[...]										
b^2	[...]										
ΔT^2	[...]										
Δb^2	[...]										

III.4. Graph

IV. SUMMARY

Since on the graph … it is possible to draw a straight line passing through all measurement uncertainty rectangles, there are no grounds to conclude a deviation from …
Alternatively:
The deviation from linearity in the range … may result from …
Additional conclusions, observations, causes of measurement uncertainties.