Theoretical Introduction

Experiment “P E N D U L U M”

The gravitational acceleration g is the acceleration with which every body would move in the gravitational field near the Earth’s surface if aerodynamic air resistance did not exist. The gravitational acceleration is equal to the gravitational field strength ***E***.

The gravitational acceleration depends on latitude. Two factors influence this: the centrifugal force resulting from the rotational motion of the Earth and the changing distance from the centre of the Earth.

Harmonic oscillations are defined as the motion of a mass $m$along the coordinate x, on which a force acts that is proportional to the value of this coordinate, with the opposite sign.

$$F = - kx$$

$$a = \frac{F}{m} \quad \text{(Newton’s second law of dynamics)}$$

where:
A– amplitude,
ω– frequency $(2\pi/T)$, argument of the sine function – phase,
φ– initial phase.
The solution turns out to be correct when: ⍰^{2} = k/m.

A mathematical pendulum is a special case of a physical pendulum. Namely, a physical pendulum becomes a mathematical pendulum if the mass of the body is concentrated at one point while remaining connected to the axis. If the oscillations are small, not exceeding approximately $5°$of deflection from the vertical, then:

$$\sin\alpha = dx \approx \alpha \quad \text{therefore:} \quad \frac{d^2\alpha}{dt^2} = \frac{-mgd\alpha}{I_0 + md^2}$$

The above differential equation has a solution in the form of harmonic oscillations with the period:

$$T = 2\pi\sqrt{\frac{I_0 + \mathrm{md}^2}{\mathrm{mgd}}}$$

As a result of concentrating the mass at point “0”, the moment of inertia is: I_0=0.

Then the expression for the period of a physical pendulum is transformed into the expression for the period of a mathematical pendulum, where the symbol d, the distance of the axis from the centre of mass, is replaced by the symbol l, the length of the pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

On the Earth’s surface, the period of a pendulum performing small oscillations depends only on its length. The period of oscillation of a mathematical pendulum is proportional to the square root of its length.

“P E N D U L U M”

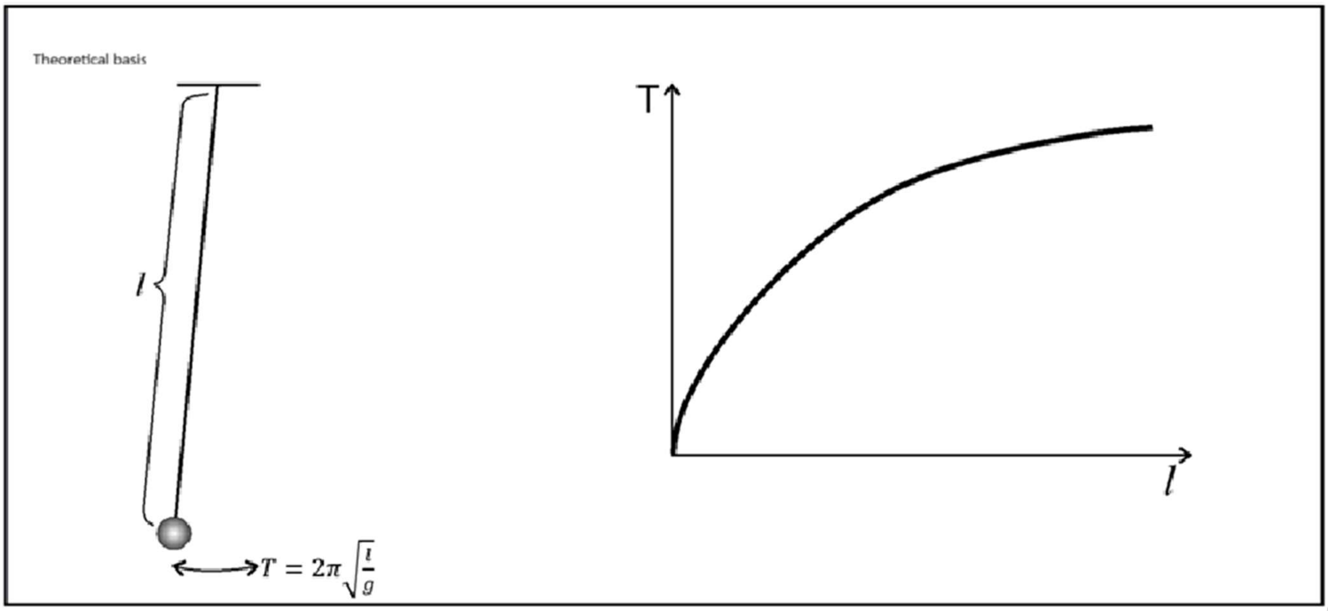
Preparation Questions:

1. What is a gravitational field?
2. Give the definition of gravitational field strength.
3. What is the value of gravitational field strength at the Earth’s surface?
4. What is the value and unit of gravitational acceleration?
5. State the law of universal gravitation.
6. What does the value of gravitational acceleration depend on?
7. What is the difference between the mass and the weight of a body?
8. What is a mathematical pendulum?
9. Give the formula for the period of oscillation of a mathematical pendulum.
10. What is a physical pendulum?
11. What is the period of oscillation?
12. What oscillations are called harmonic?
13. What is the gravitational acceleration on a planet with a diameter twice smaller than the Earth’s but with the same mass?
14. What is the gravitational acceleration on a planet with twice the mass of the Earth but with the same radius?
15. What is the approximate gravitational acceleration on the Moon and on Mars?

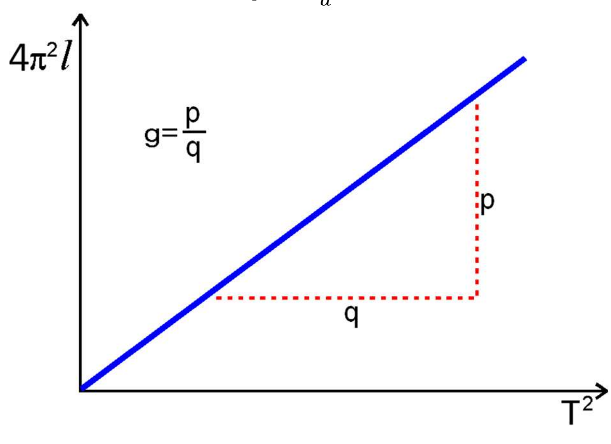
“P E N D U L U M”

<u>Szablon metodyczny</u>

Student 1: Determination of gravitational acceleration using a mathematical pendulum.
Student 2: Verification of the dependence of the period of a mathematical pendulum on its length.

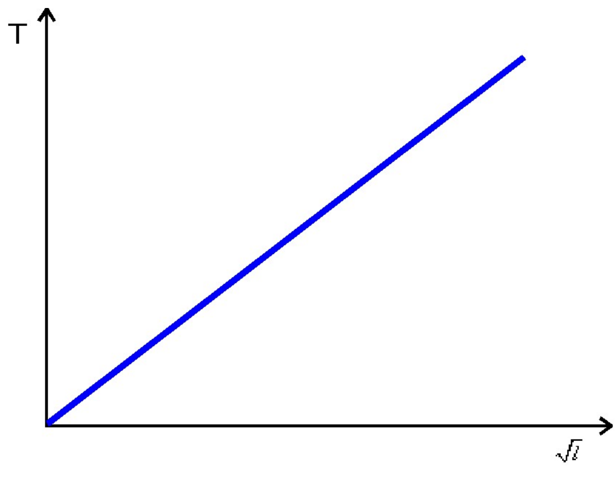


$$\underbrace{4\pi^2 l}_{Y} = \underbrace{g}_{a} \cdot \underbrace{T^2}_{X}$$



Therefore, in order to determine the gravitational acceleration, one should:

- perform measurements of the duration of ten periods of oscillation of the pendulum for ten different lengths,
- prepare a graph of the dependence:
  $4\pi^2 l$ as a function of: T^2
- read from it the value of the gravitational acceleration *g.*



Therefore, in order to verify the dependence of the period of the pendulum on its length, one should:

- perform measurements of the duration of ten periods of oscillation of the pendulum for ten different lengths,
- prepare a graph of the dependence of the pendulum period T on the square root of the length,
- analyse its linearity

“P E N D U L U M”

Report Guidelines – Determination

Student 1: Determination of gravitational acceleration using a mathematical pendulum.

I. **Methodology** (conceptual plan of the exercise)
II. **Course of the exercise**
   II.1. Course of activities
   II.2. Sketch of the measurement setup
III. **Results**

III.1. Measurement results

		1	2	3	4	5	6	7	8	9	10
l	[m]										
t = 10 T	[s]										

Δl = ...
Δt = ... $\Delta T = \Delta t/10$ = ...

III.2. Calculations (example calculations — referring, for example, to measurement no. 4)

T^2 = ...

$4\pi^2 l$ = ...

$\Delta(T^2) = \left| T^2 - (T + \Delta T)^2 \right|$ = ...

$\Delta(4\pi^2 l) = \left| 4\pi^2 \Delta l \right|$ = ...

III.3. Calculation results

		1	2	3	4	5	6	7	8	9	10
$4\pi^2 l$	[...]										
T^2	[...]										
$\Delta(4\pi^2 l)$	[...]										
$\Delta(T^2)$	[...]										

III.4. Graph

- calculation of g, the slope of the “best-fit” straight line
- calculation of g', the slope of the deviated straight line
- calculation of: $\Delta g = |g - g'|$

IV. Summary

The determined value of … is …
Accuracy of the method: …
Additional conclusions, observations, causes of measurement uncertainties.

“P E N D U L U M”

Report Guidelines – Verification

Student 2: Verification of the dependence of the period of a mathematical pendulum on its length.

I. **Methodology** (conceptual plan of the exercise)
II. **Course of the exercise**
II.1. Course of activities
II.2. Sketch of the measurement setup
III. **Results**

III.1. Measurement results

		1	2	3	4	5	6	7	8	9	10
l	[m]										
t = 10 T	[s]										

Δl = ...
Δt = ...

III.2. Calculations (example calculations — referring, for example, to measurement no. 4)

$T = t/10 = \ldots$

$\Delta T = \Delta t/10 = \ldots$

$\sqrt{l} = \ldots$

$\Delta\sqrt{l} = \left|\sqrt{l} - \sqrt{l + \Delta l}\right| = \ldots$

III.3. Calculation results

		1	2	3	4	5	6	7	8	9	10
T	[...]										
$\sqrt{l}$	[...]										
$\Delta\sqrt{l}$	[...]										

ΔT = ...

III.4. Graph

IV. Summary

Since on the graph … it is possible to draw a straight line passing through all measurement uncertainty rectangles, there are no grounds to conclude a deviation from …
Alternatively:
The deviation from linearity in the range … may result from …
Additional conclusions, observations, causes of measurement uncertainties.